Singularities of particle trajectory caustics and beam shaping in bunch compressors

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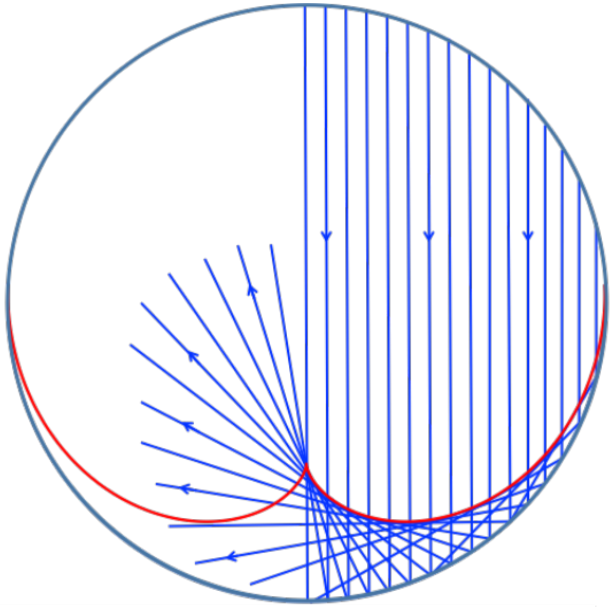
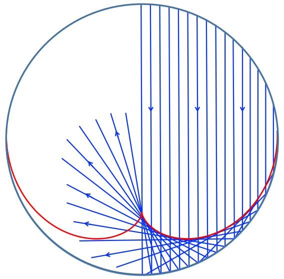
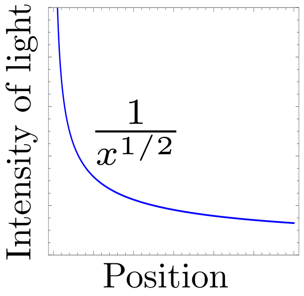
Caustics



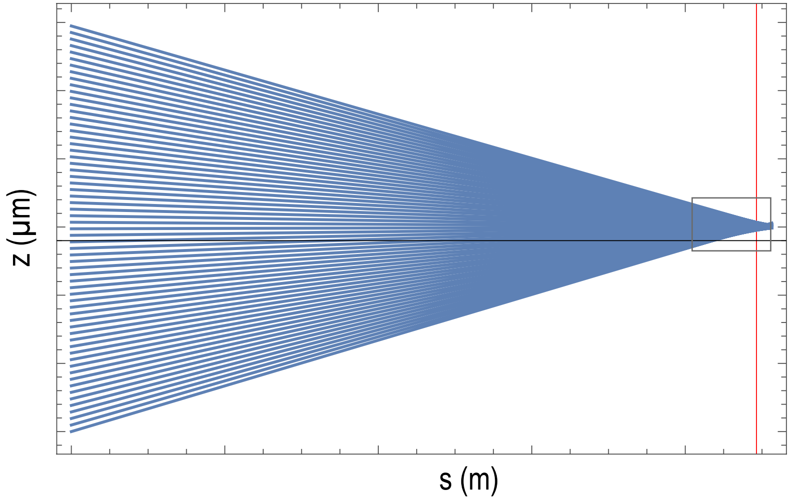
Caustics

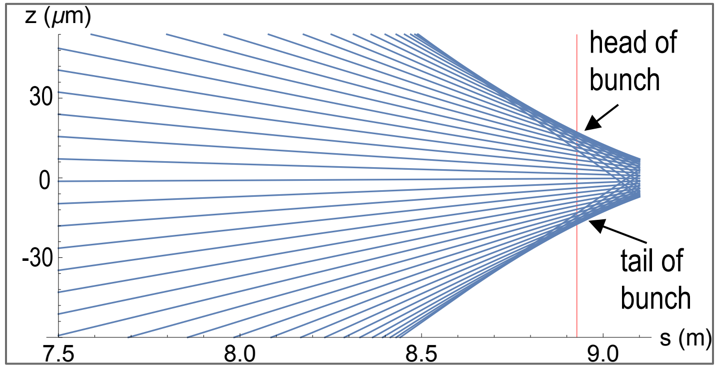
Optical caustics:



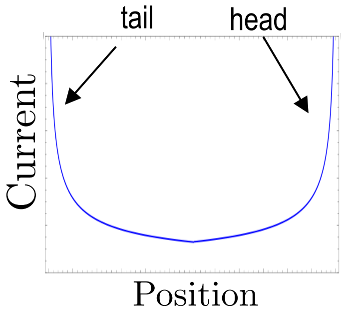


Electron trajectories caustics:





These current horns are a manifestation of the caustic nature of the electron trajectories, as singularities of families of trajectories are encountered.



Caustics / Catastrophe theory

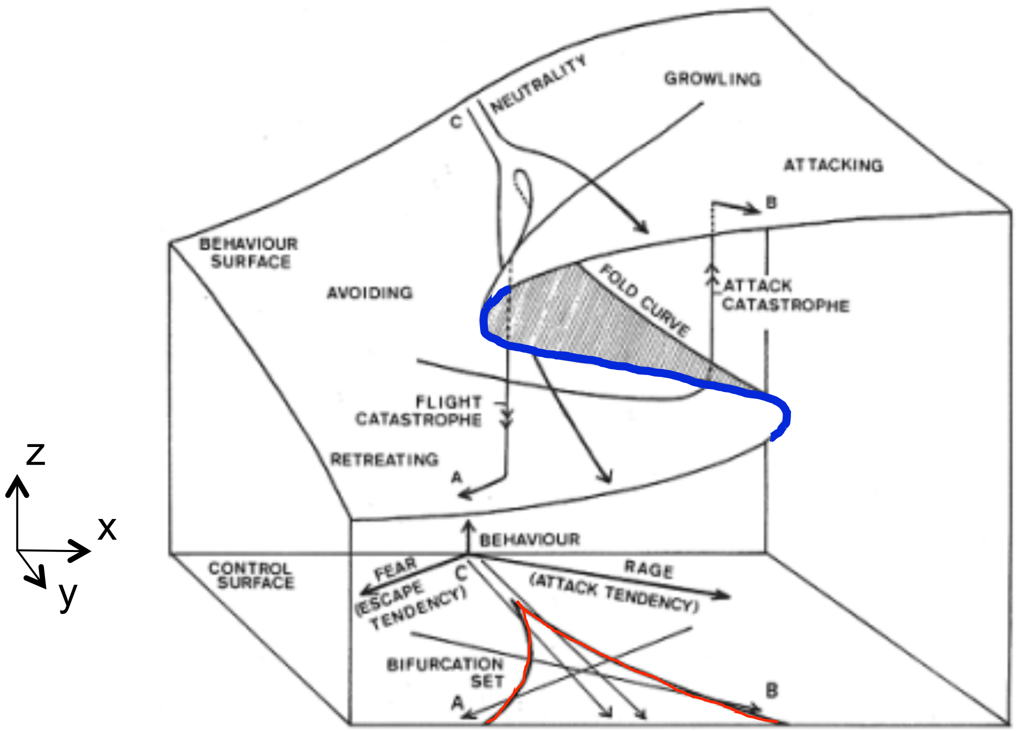
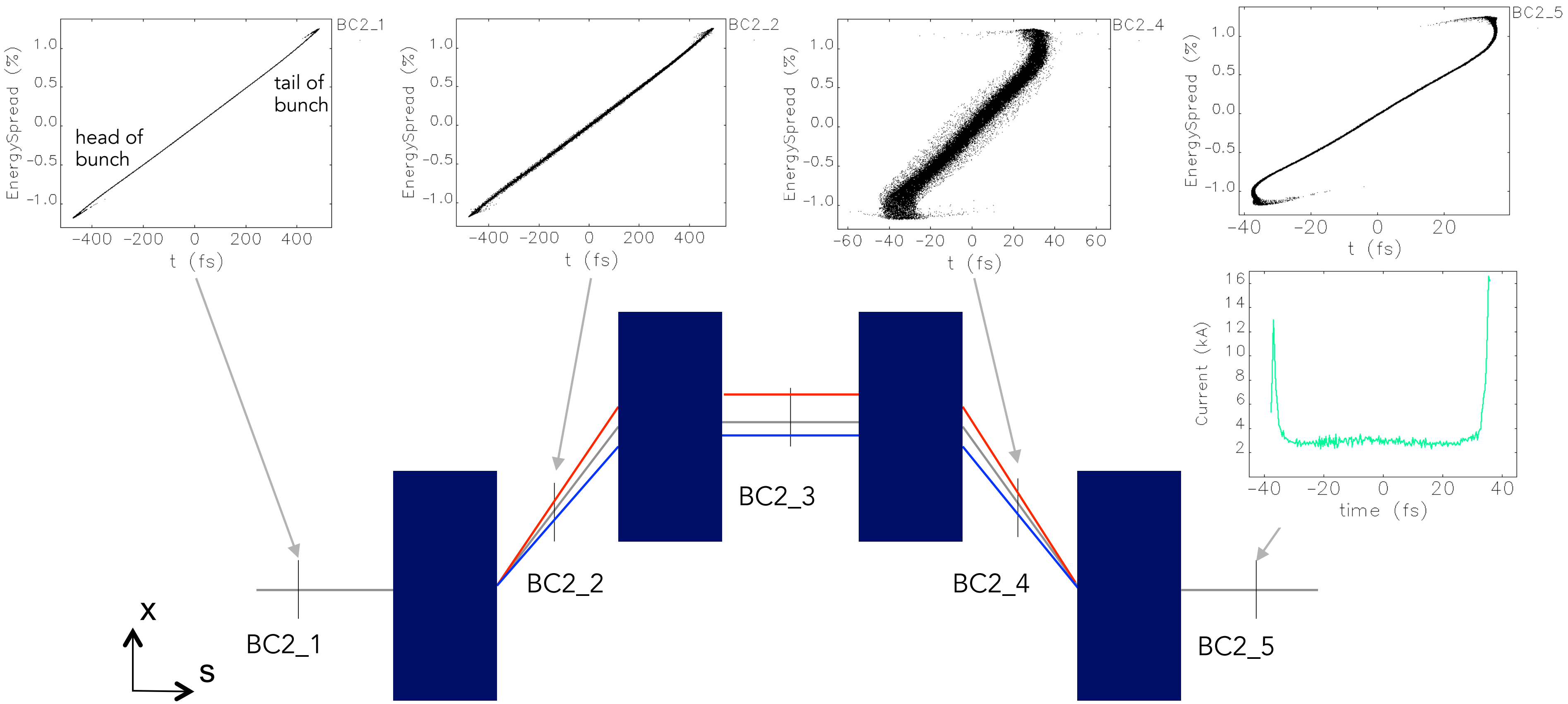


Image: EC Zeeman (1976) Catastrophe Theory in Scientific American.

Current horns

- Current horns are resultant from caustics



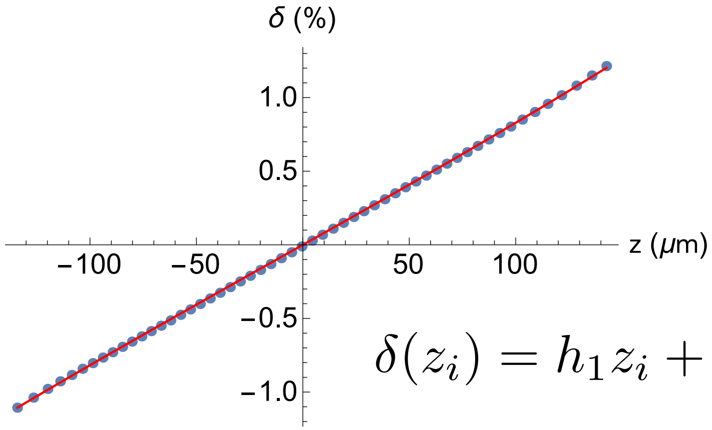
Caustic expression

- Expression for caustics:

$$\tilde{z}(z_i) = z_i + \frac{\delta(z_i)(-1 + T_{566}(-2 + \delta(z_i))\delta'(z_i) + U_{5666}(-3 + \delta(z_i)^2)\delta'(z_i))}{\delta'(z_i)}$$

$$\tilde{R_{56}}(z_i) = \frac{-1 - 2T_{566}\delta'(z_i) - 3U_{5666}\delta'(z_i)}{\delta'(z_i)}$$

Initial longitudinal phase space distribution:



$$\delta(z_i) = h_1 z_i + h_2 z_i^2 + h_3 z_i^3$$

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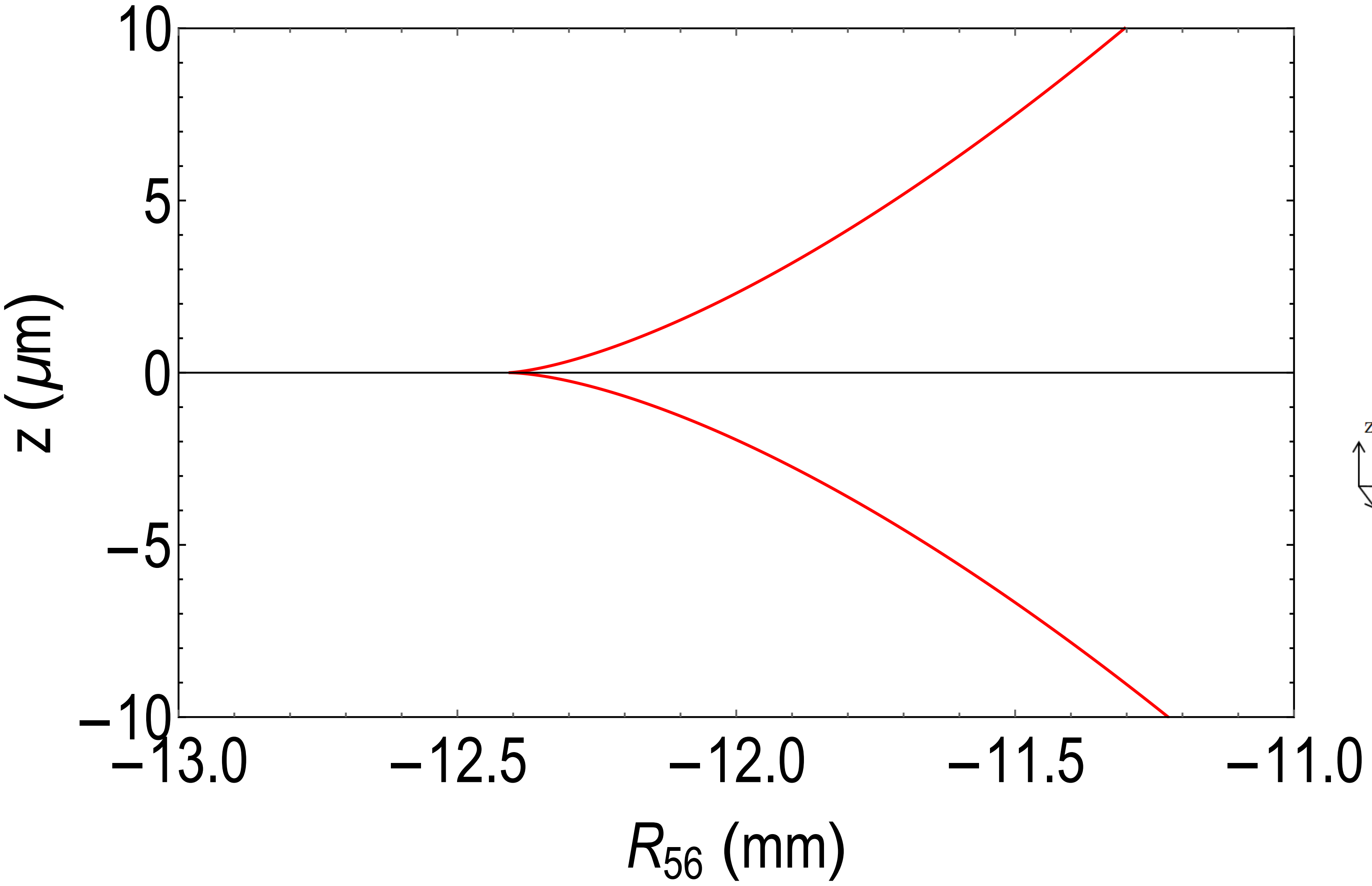
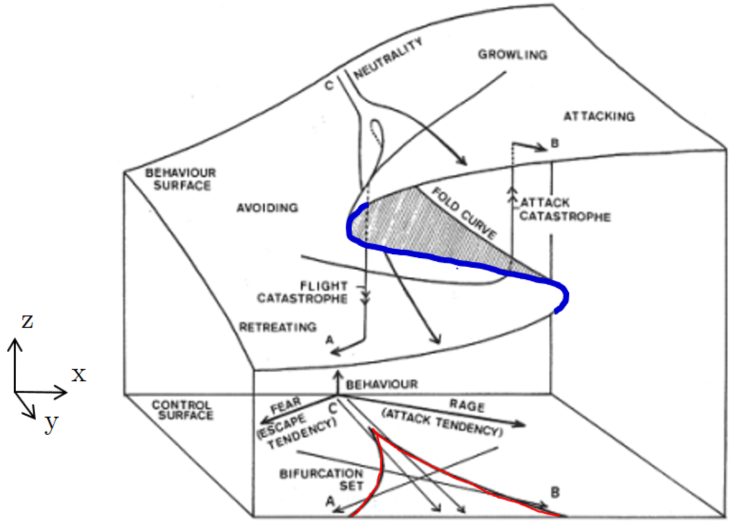
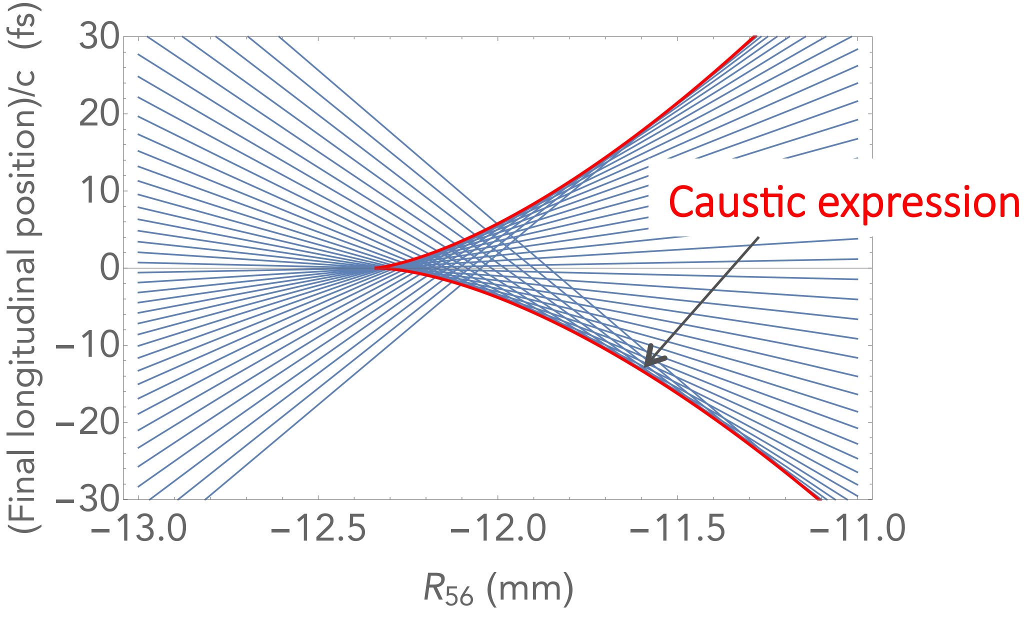
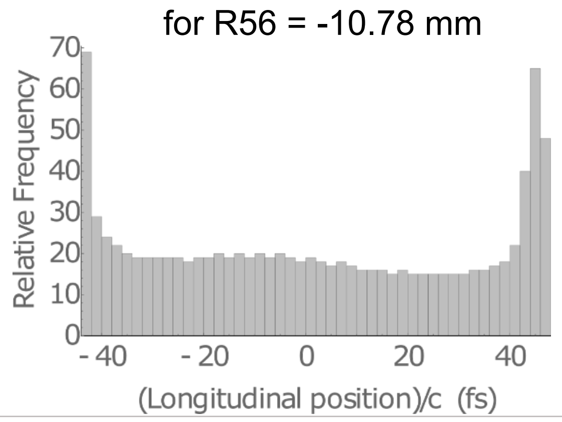
Caustic expression



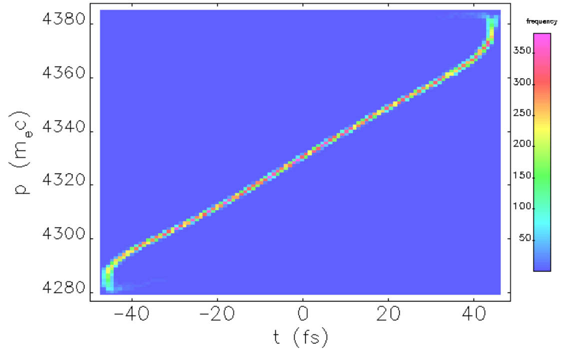


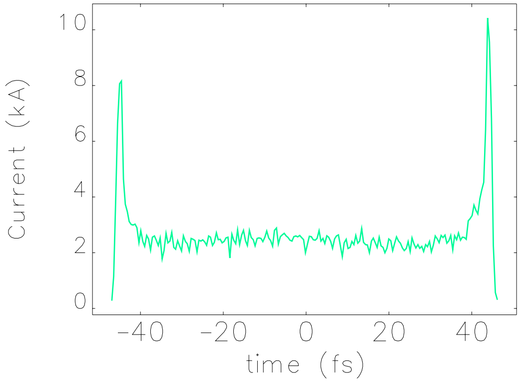
Comparison with simulations





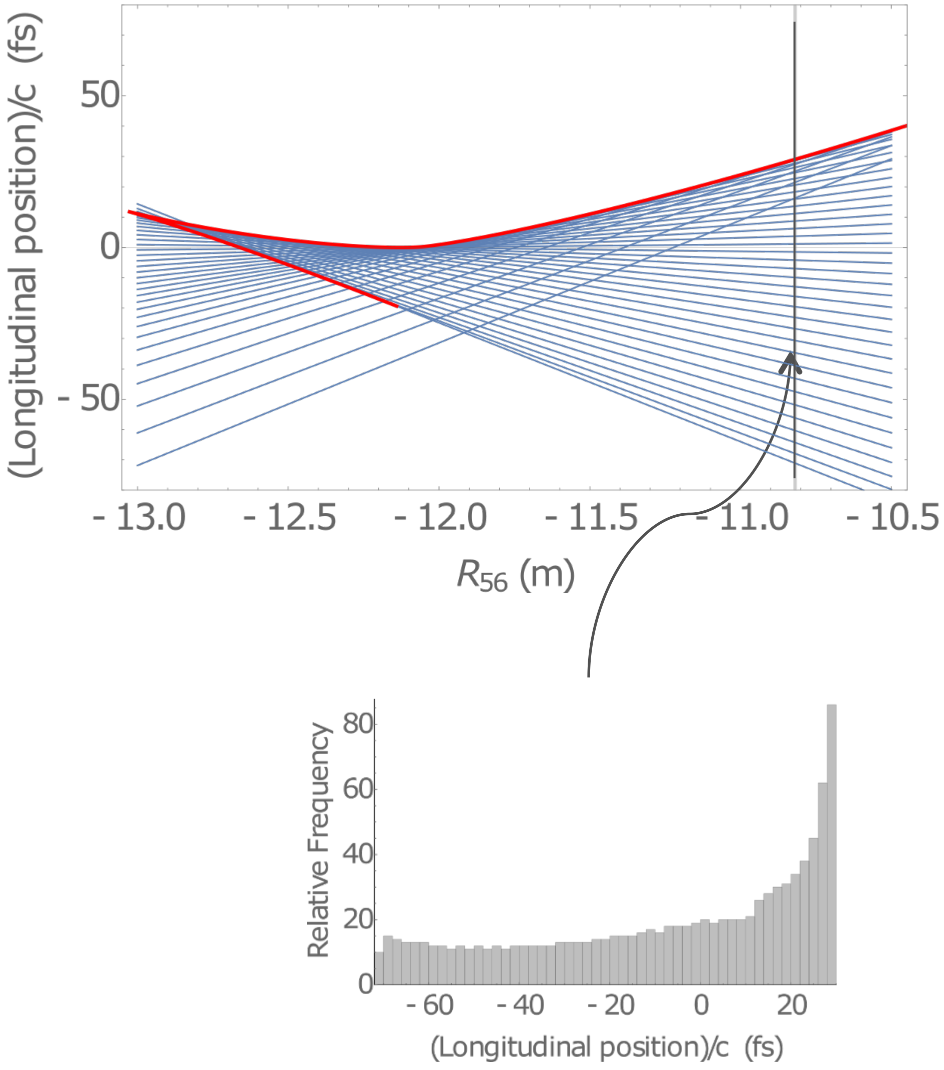
ELEGANT simulation results:



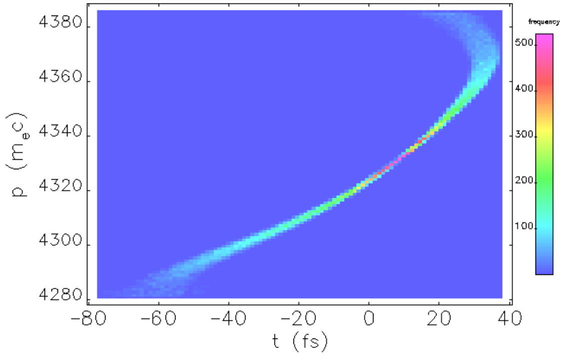


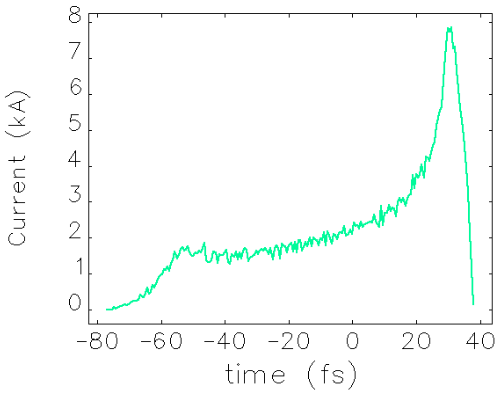
Double–horned current structure produced at the end of a dispersive region where R_{56} = -10.78 mm, T_{566} = 16.35 mm and U_{5666} = -11.38 mm.

Comparison with simulations



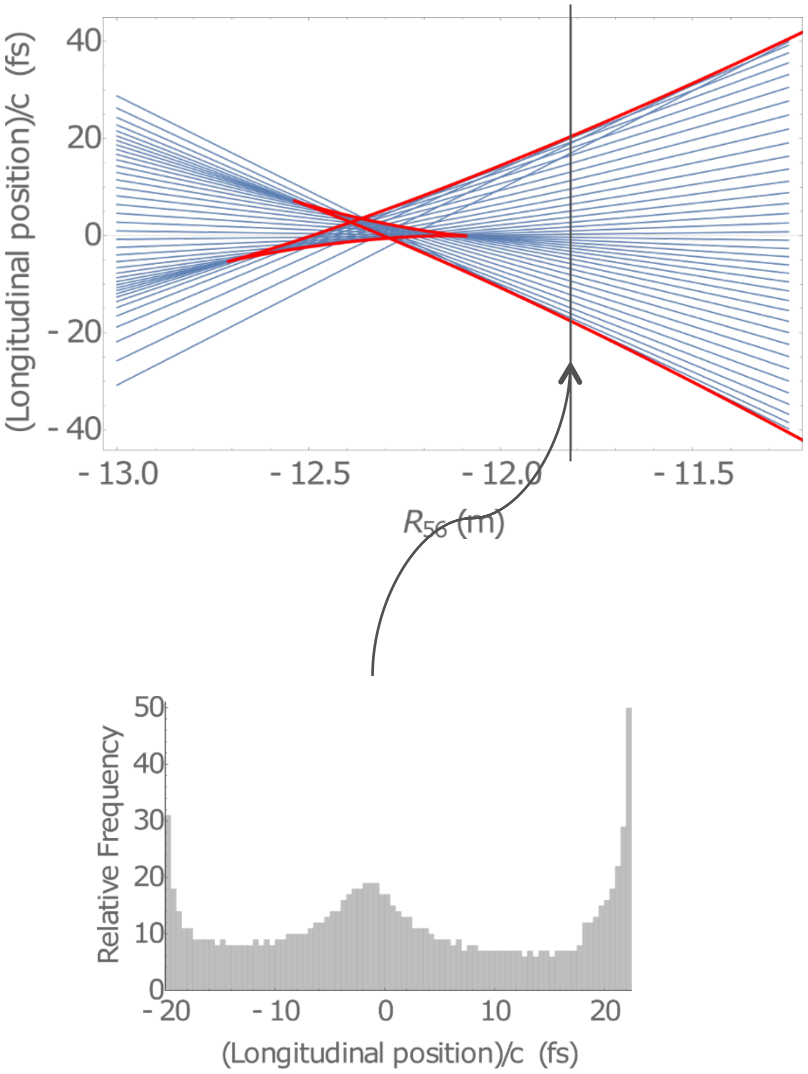
ELEGANT simulation results:



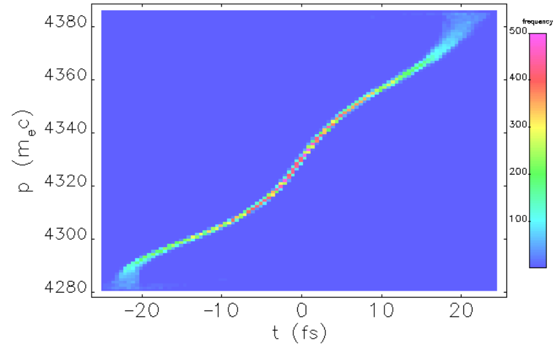


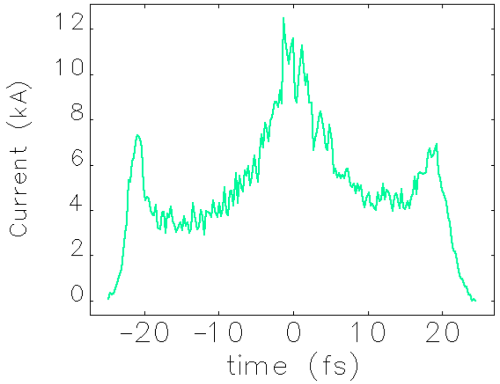
Single–horn current profile produced with R_{56} = -10.82 mm, T_{566} = -41.07 mm and U_{5666} = 0.40 m

Comparison with simulations



ELEGANT simulation results:





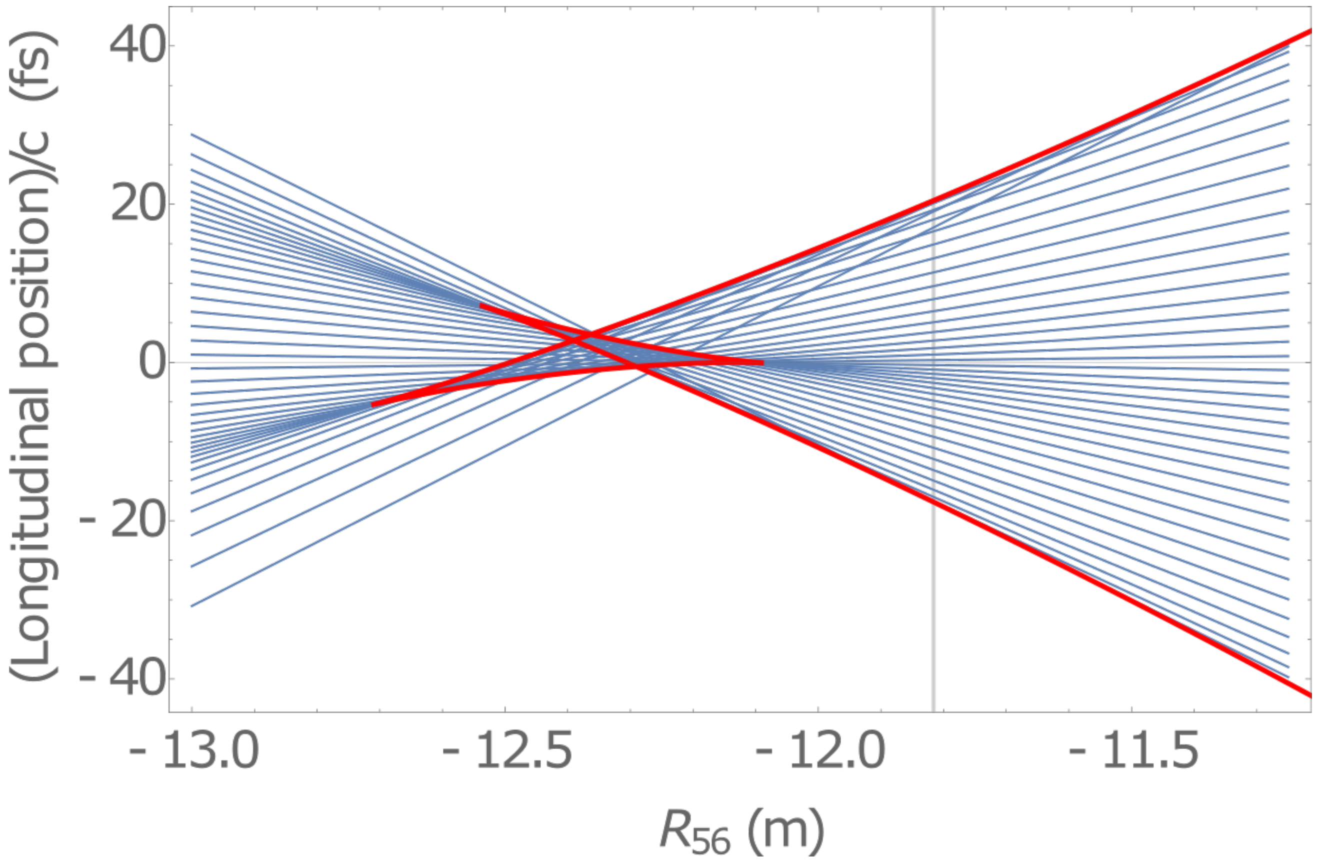
Current profile produced with R_{56} = -11.76 mm, T_{566} = 16.10 mm and U_{5666} = 2.60 m.

A closer look…

$$\tilde{z}(z_i) = z_i + \frac{\delta(z_i)(-1 + T_{566}(-2 + \delta(z_i))\delta'(z_i) + U_{5666}(-3 + \delta(z_i)^2)\delta'(z_i))}{\delta'(z_i)}$$

$$\tilde{R_{56}}(z_i) = \frac{-1 - 2T_{566}\delta'(z_i) - 3U_{5666}\delta'(z_i)}{\delta'(z_i)}$$

Three types of catastrophes but all described by the same parametric expression.



Can we avoid caustic current horns?

Caustic condition

Caustics condition:

$$R_{56}s\frac{d(\delta(z))}{dz}\Bigg|_{z=z_i} + T_{566}s\frac{d}{dz}\left(\delta^2(z)\right)\Bigg|_{z=z_i} + U_{5666}s\frac{d}{dz}\left(\delta^3(z)\right)\Bigg|_{z=z_i} + s_{BC} = 0.$$

where,

δ = energy spread
z_i = initial longitudinal position
s = position along accelerator
s_{BC} = end of bunch compressor position

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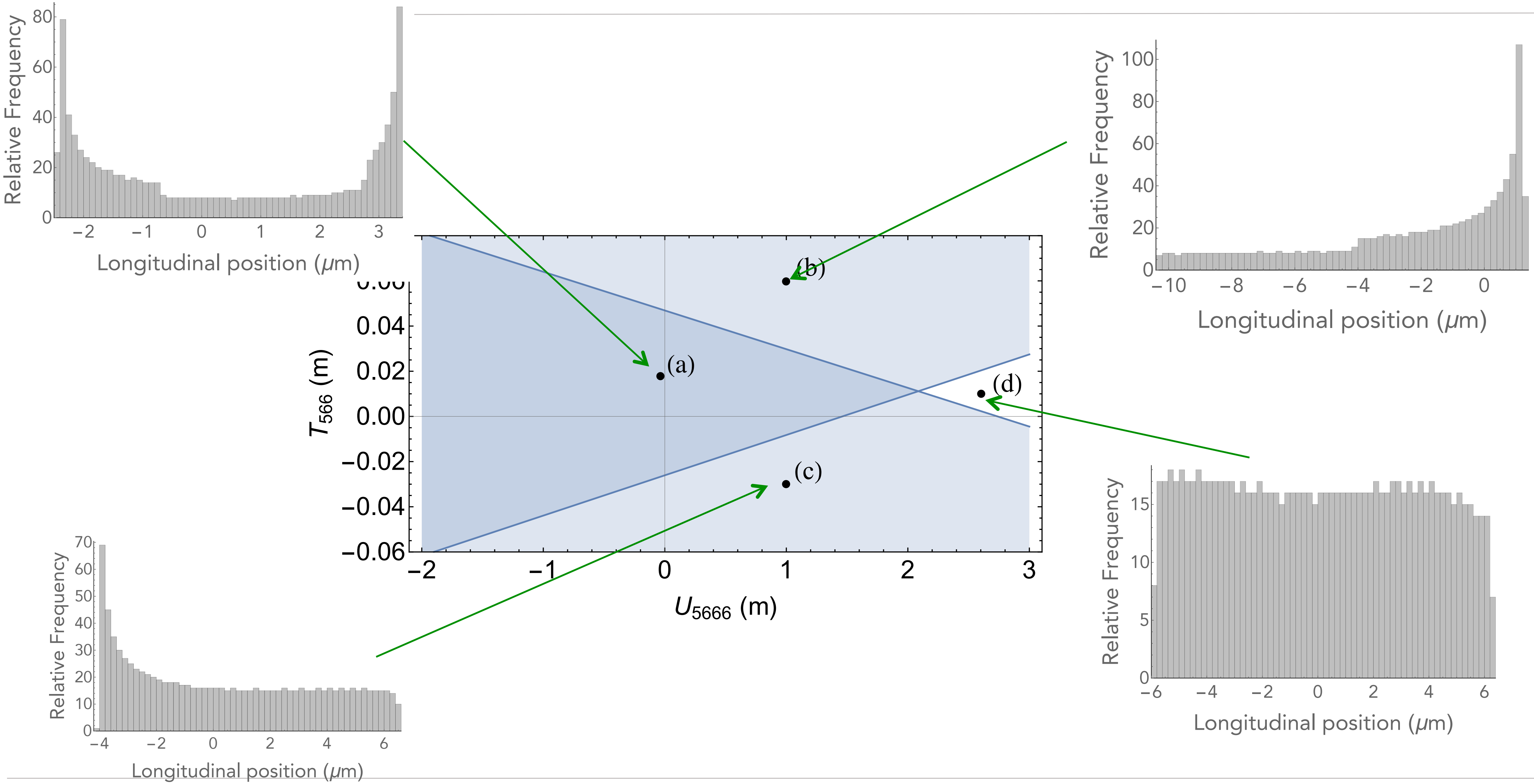
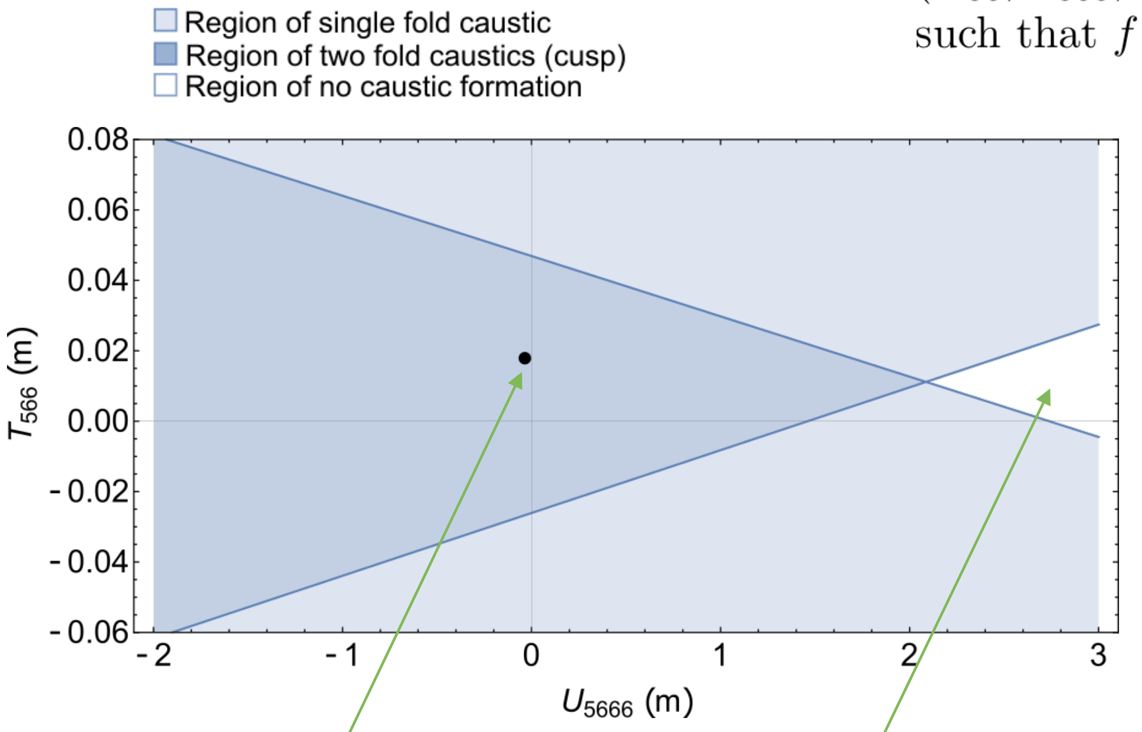
Avoiding caustics



Boundaries between caustic regions

The border between the caustic regions is defined by the set of variables, $(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3)$, evaluated for a given initial bunch with $z_{\mathrm{min/max}}$, such that $f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3\,;\, z_{\mathrm{min/max}}) = 0$, with,



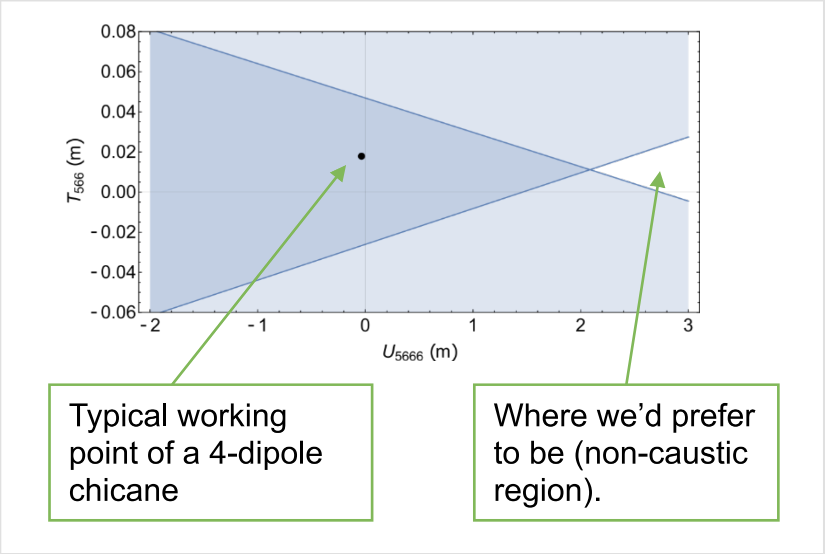
$$
\begin{aligned}
f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3; z_{\mathrm{min/max}}) = \\
1 + h_1 R_{56} + 2h_2 R_{56} z_{\mathrm{min/max}} \\
+ 3h_3 R_{56} z^2_{\mathrm{min/max}} + 2T_{566} h_1^2 z_{\mathrm{min/max}} \\
+ 6T_{566} h_1 h_2 z^2_{\mathrm{min/max}} + 3h_1^3 U_{5666} z^2_{\mathrm{min/max}}.
\end{aligned}
$$

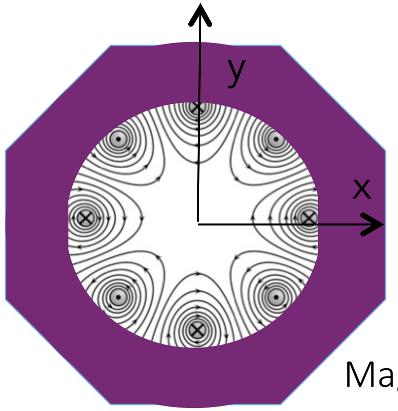
Typical working point of a 4-dipole chicane

Where we'd prefer to be (non-caustic region).

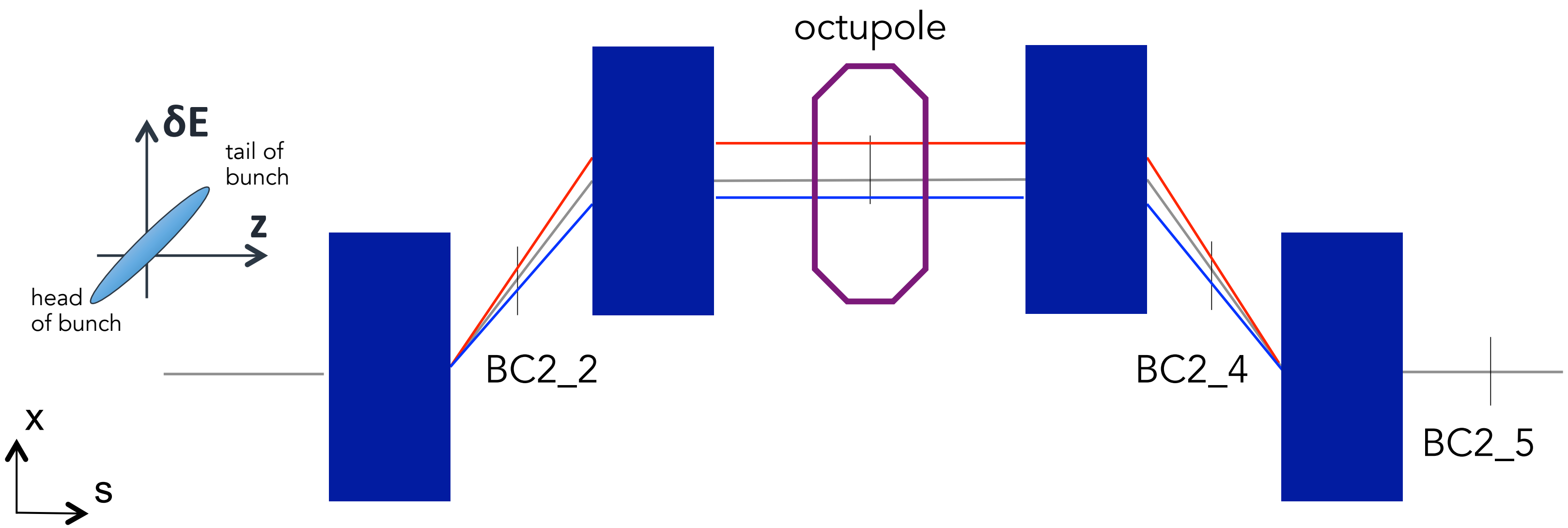
Considering a BC where the entrance bunch is characterized by h_1, h_2, and h_3 (i.e. the first, second and third order chirp), for a given value of R_{56}, the values of T_{566} and U_{5666} can be chose to ensure the working point is within the non-caustic region.

Using octupoles to alter U5666



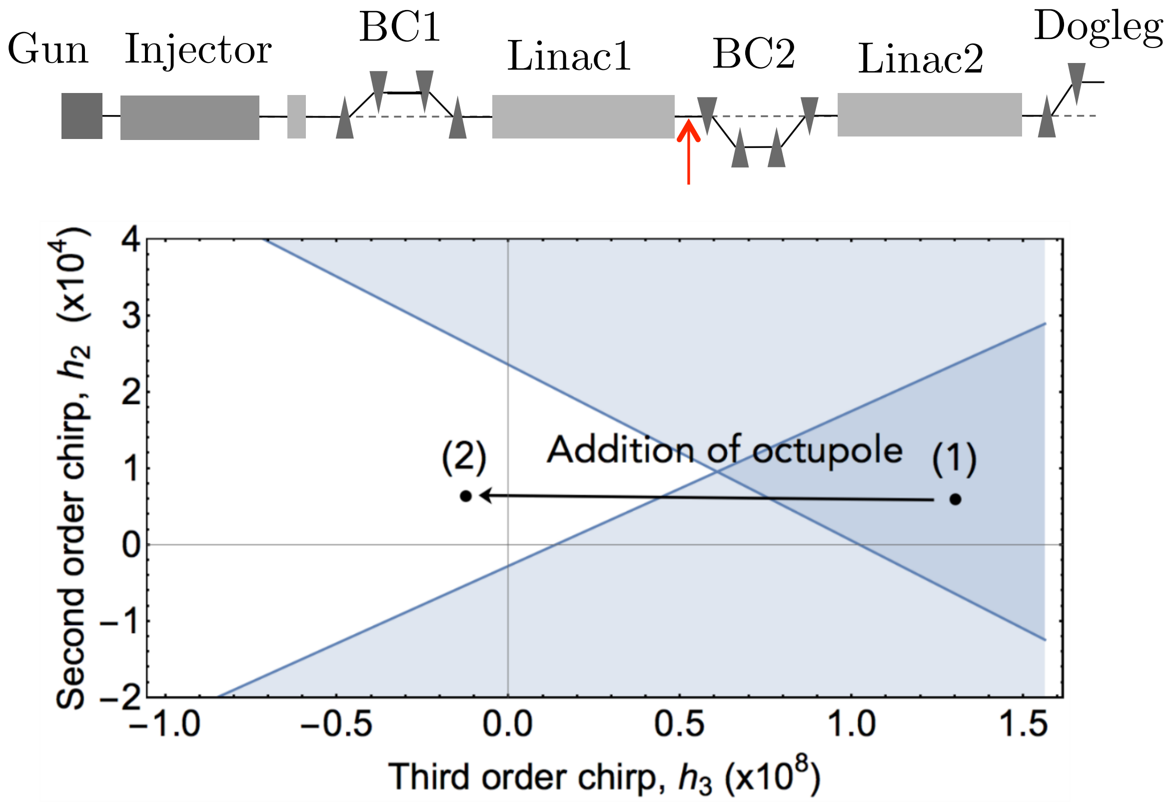


Magnetic field varies as x^3 and y^3



Boundaries between caustic regions

The border between the caustic regions is defined by the set of variables, $(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3)$, evaluated for a given initial bunch with $z_{\min/\max}$, such that $f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3\ ;\ z_{\min/\max}) = 0$, with,



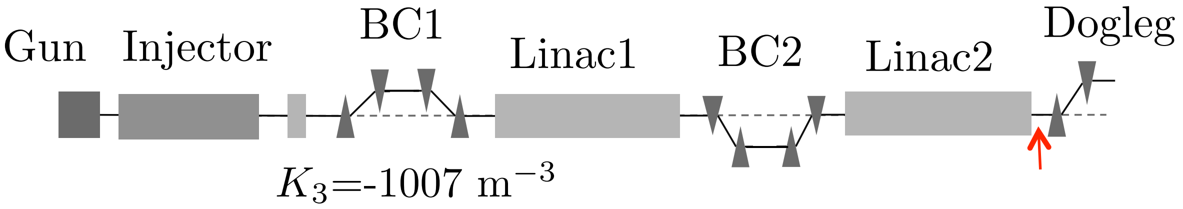
$$f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3; z_{\min/\max}) = \\ 1 + h_1 R_{56} + 2h_2 R_{56} z_{\min/\max} \\ + 3h_3 R_{56} z^2_{\min/\max} + 2T_{566} h_1^2 z_{\min/\max} \\ + 6T_{566} h_1 h_2 z^2_{\min/\max} + 3h_1^3 U_{5666} z^2_{\min/\max}.$$

Two cases were investigated:

1. Predominately **S-band** linac
2. Predominately **X-band** linac

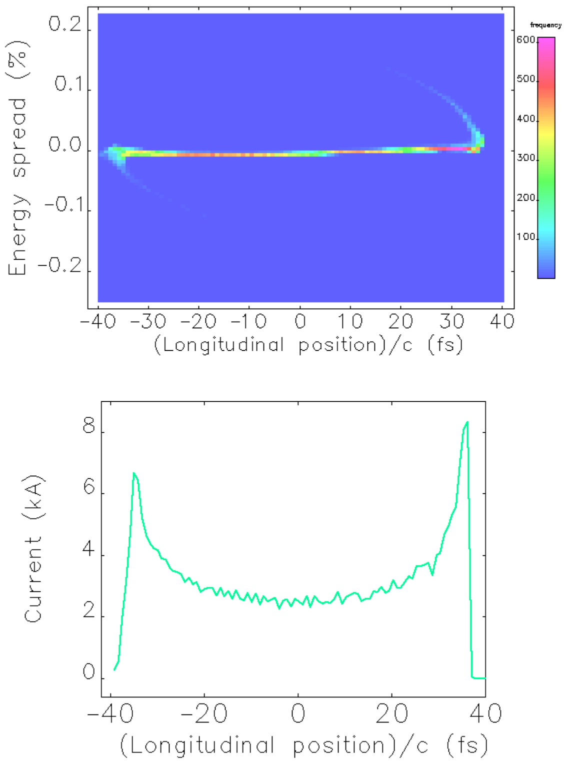
FEL, S-band linac simulations

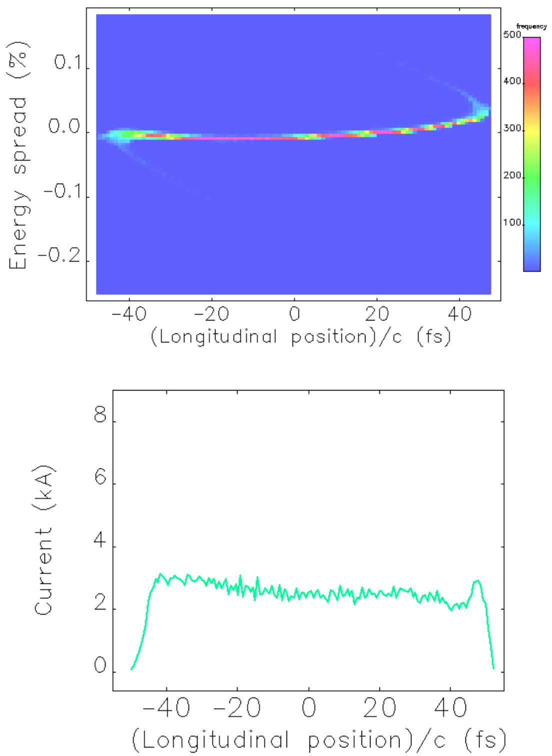
S-band Linac



Without octupole:

With octupole:





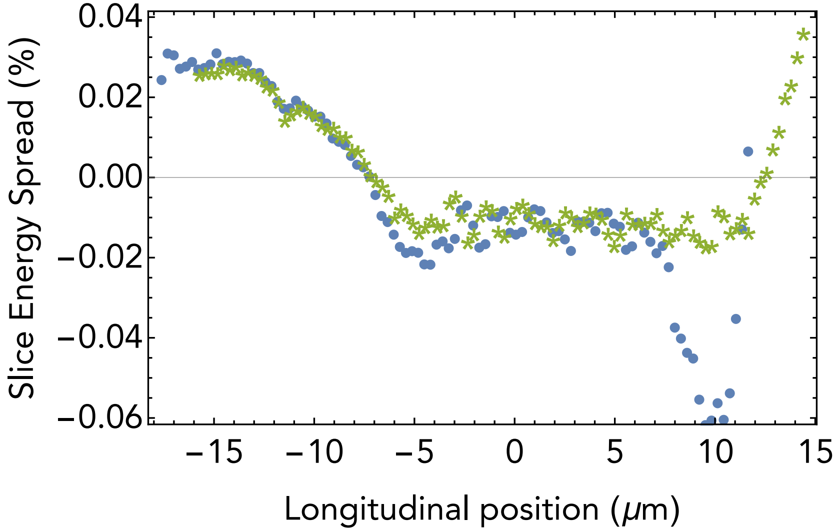
The caustic current-horns have been prevented from forming.

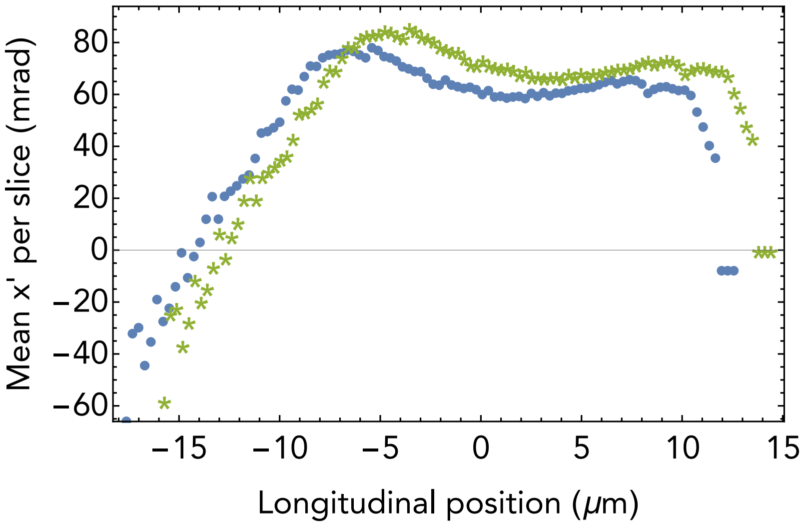
Slice properties

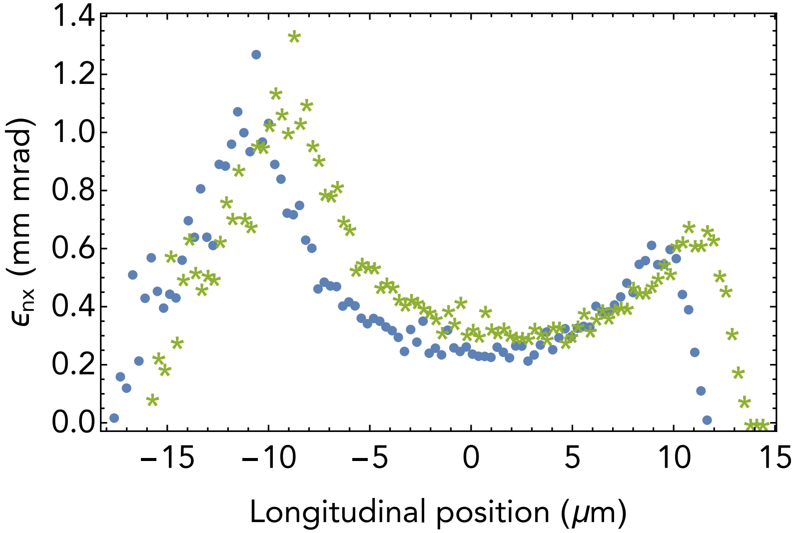
S-band linac

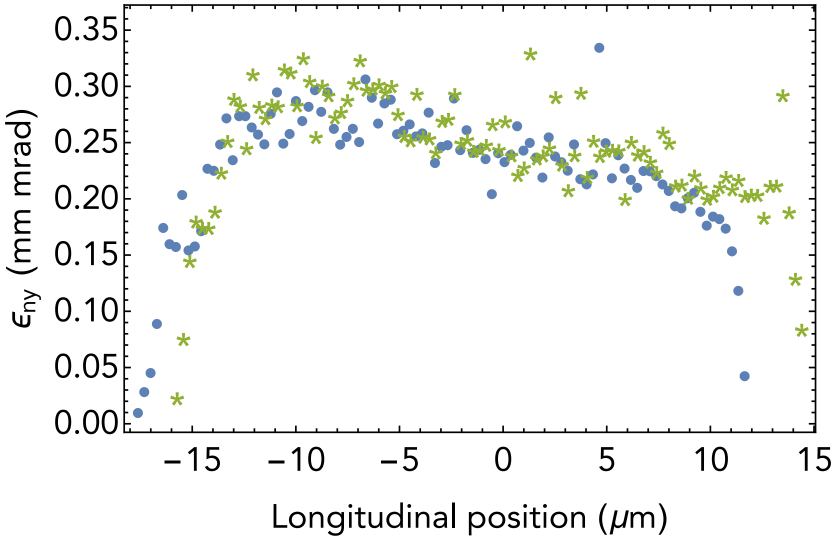
Without octupole: •

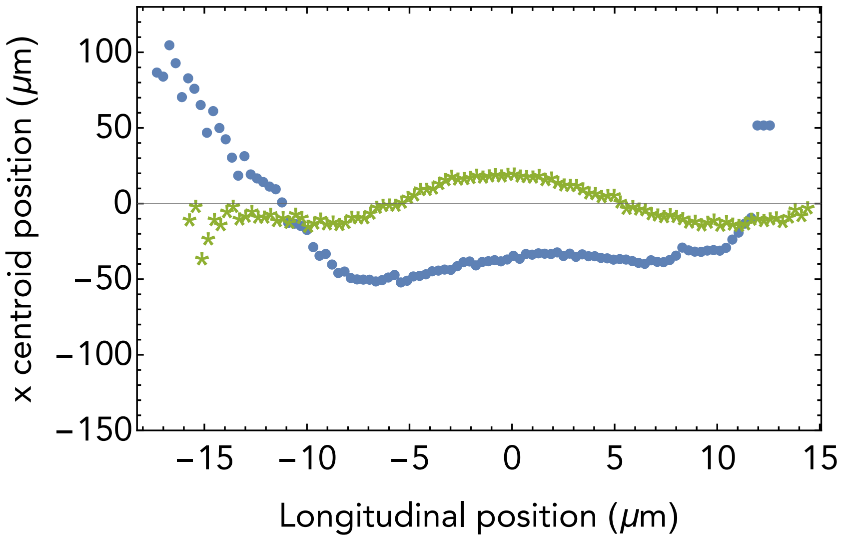
With octupole: *





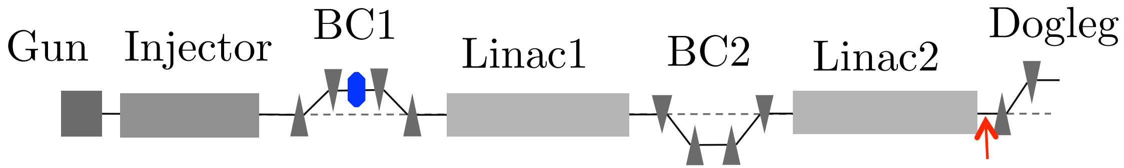






Avoiding caustics, X-band linac

X-band Linac

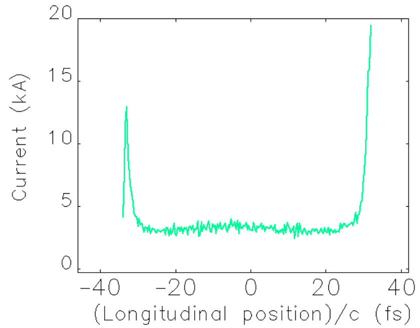


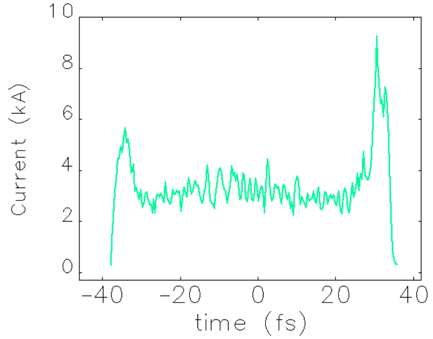
R_{56} = -82.38 mm
T_{566} = 102.40 mm
U_{5666} = -5.23 m

R_{56} = -11.18 mm
T_{566} = 16.77 mm
U_{5666} = -11.10 mm

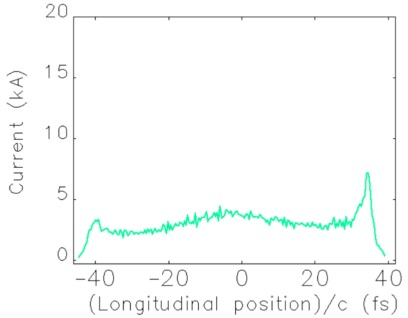
Without CSR: | With CSR:

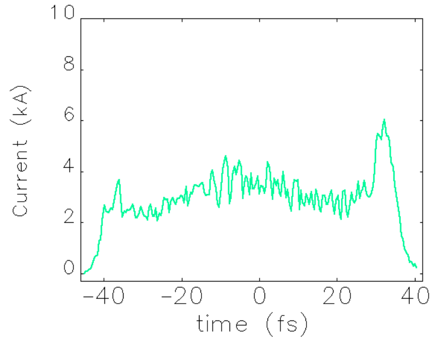
Without octupole:





With octupole:

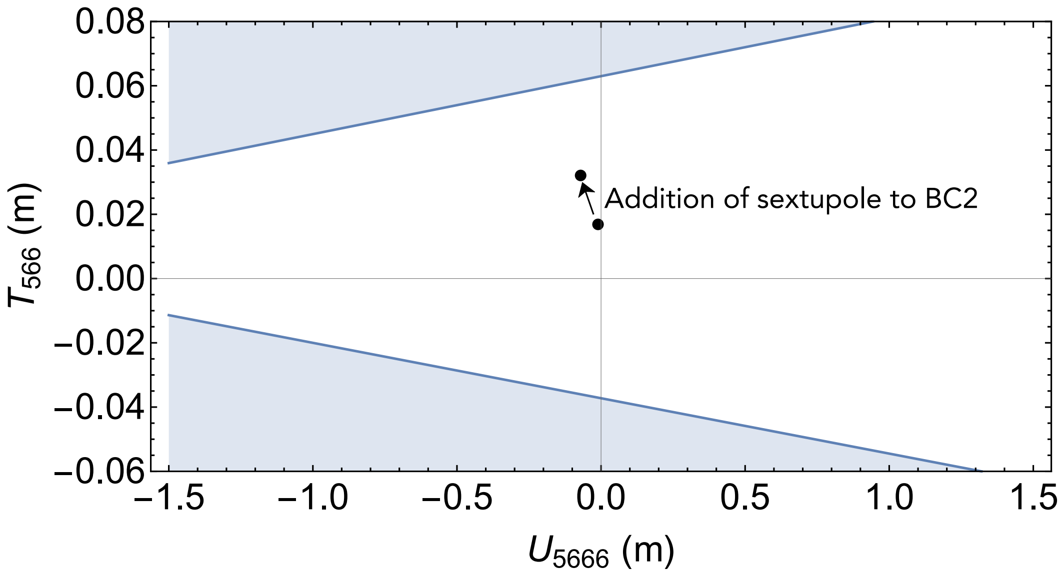




Further optimisation

R_{56} = -82.38 mm
T_{566} = 102.40 mm
U_{5666} = -5.23 m

R_{56} = -11.18 mm
T_{566} = 32.05 mm
U_{5666} = -72.00 mm

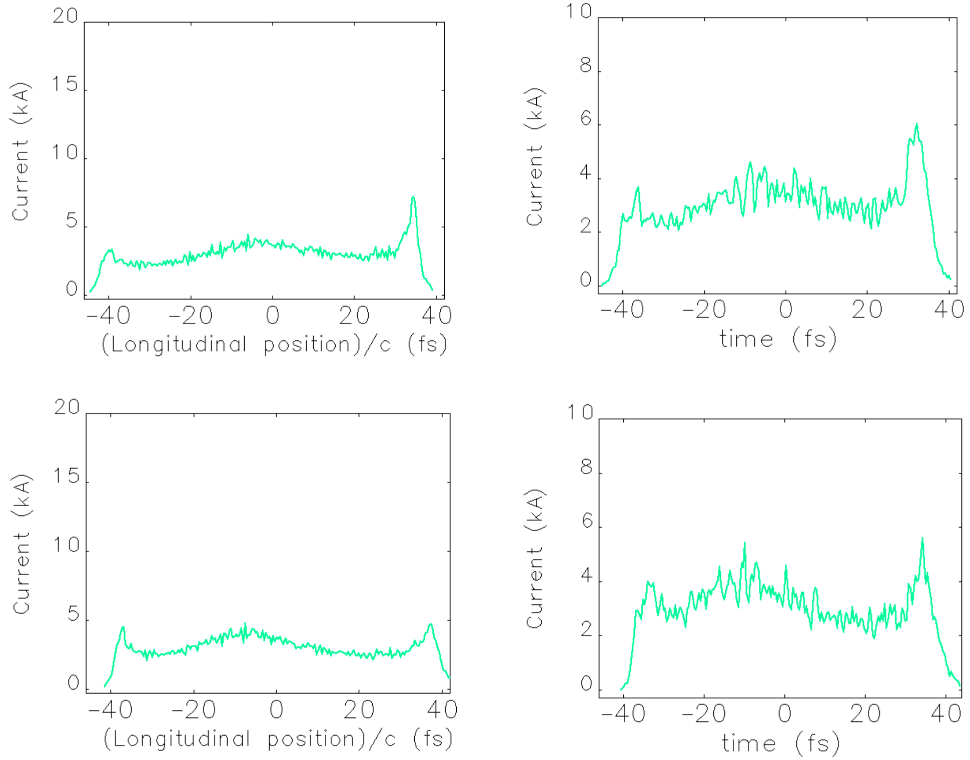


Without CSR:

With CSR:

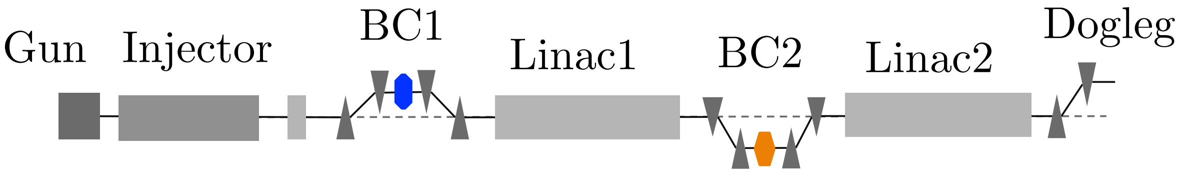
Without sextupole:

With sextupole:

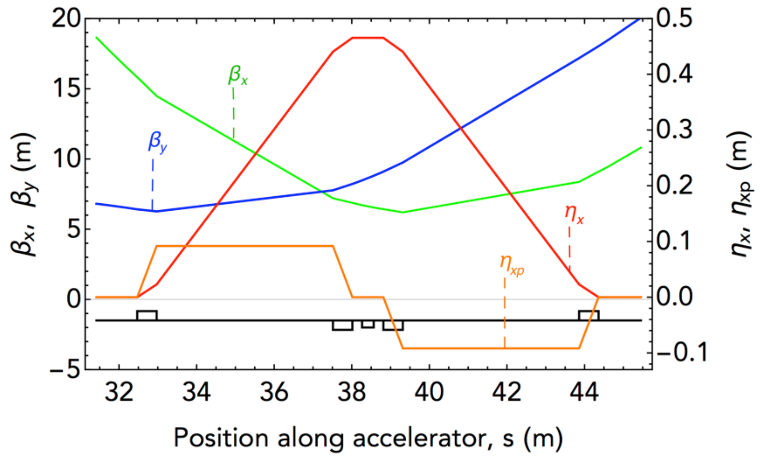


FEL, X-band linac

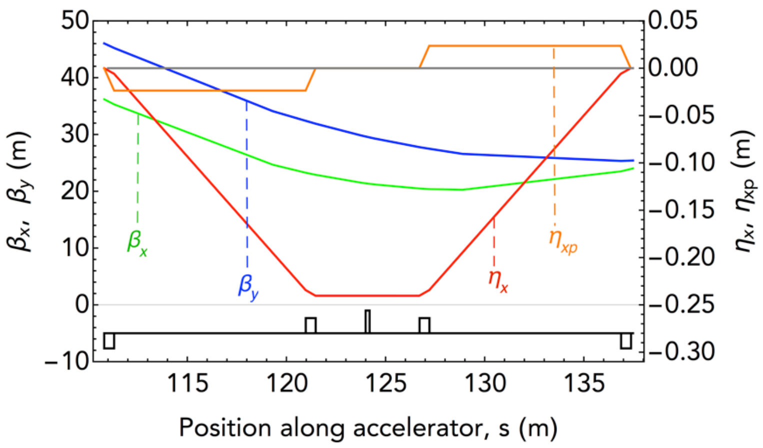
X-band Linac



Bunch Compressor 1



Bunch Compressor 2



BC1:

R56 = -82.36 mm

T566 = 102.40 mm

U5666 = -5.23 m

Dipole bending angle, $\theta = 5.25°$

Octupole strength, K3 = 2061 m^{-3}

BC2:

R56 = -11.18 mm

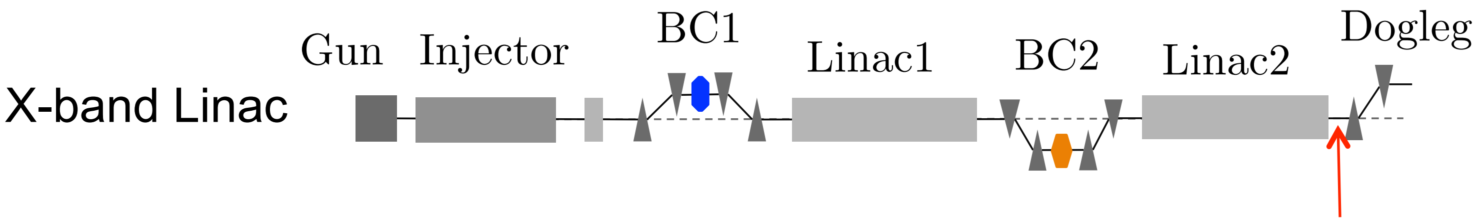
T566 = 32.10 mm

U5666 = -72.19 mm

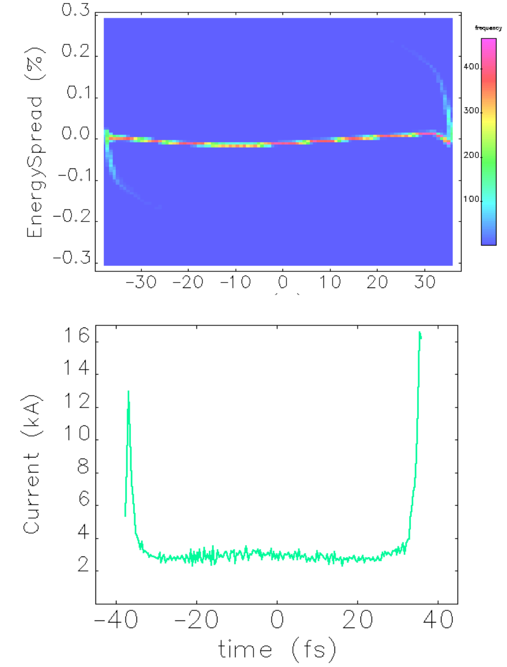
Dipole bending angle, $\theta = 1.35°$

Sextupole 1 strength, K2 = 11.03 m^{-2}

FEL, X-band linac

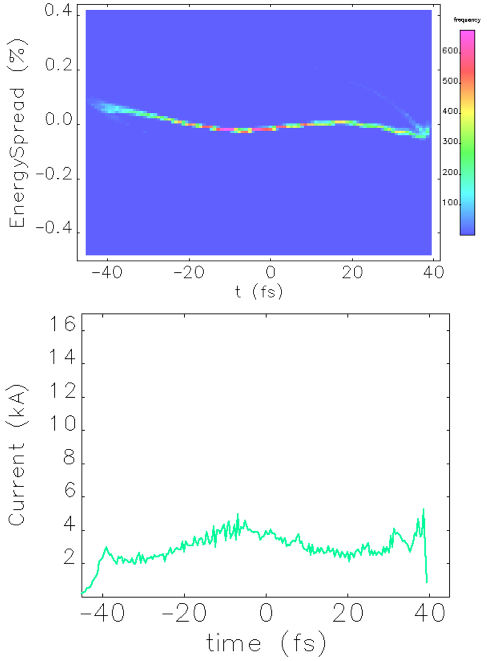


Without octupole & sextupole:



$\epsilon_{nx} = 1.394$ mm mrad

With octupole & sextupole:



$\epsilon_{nx} = 0.842$ mm mrad

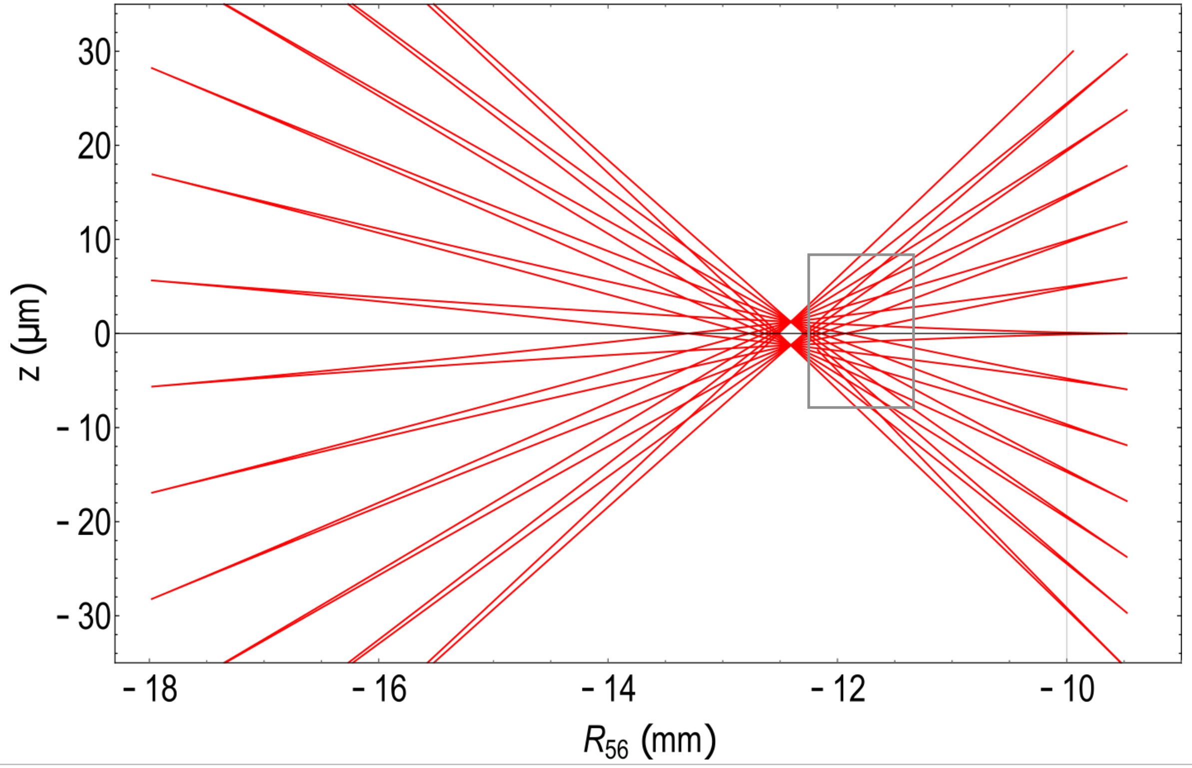
49% reduction in the CSR-induced emittance growth.

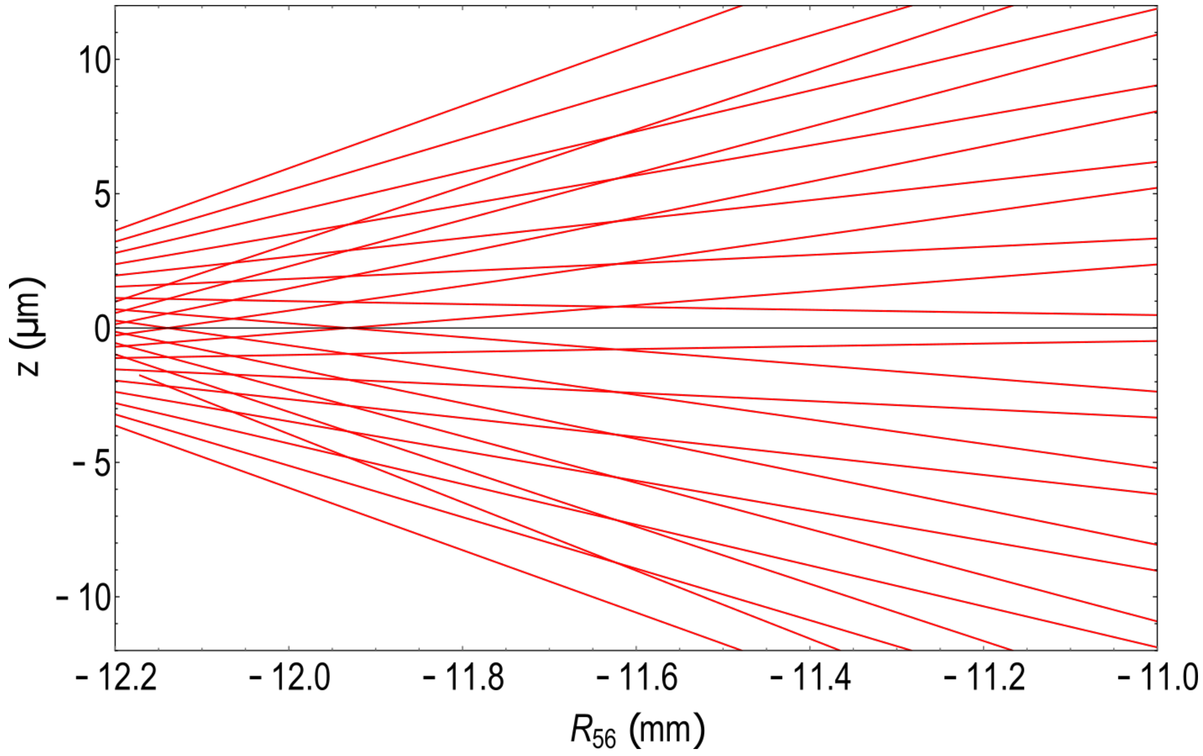
T. K. Charles et al. (2017) Phys. Rev. AB, **30**, 030705

Microbunching
– also a caustic phenomenon?

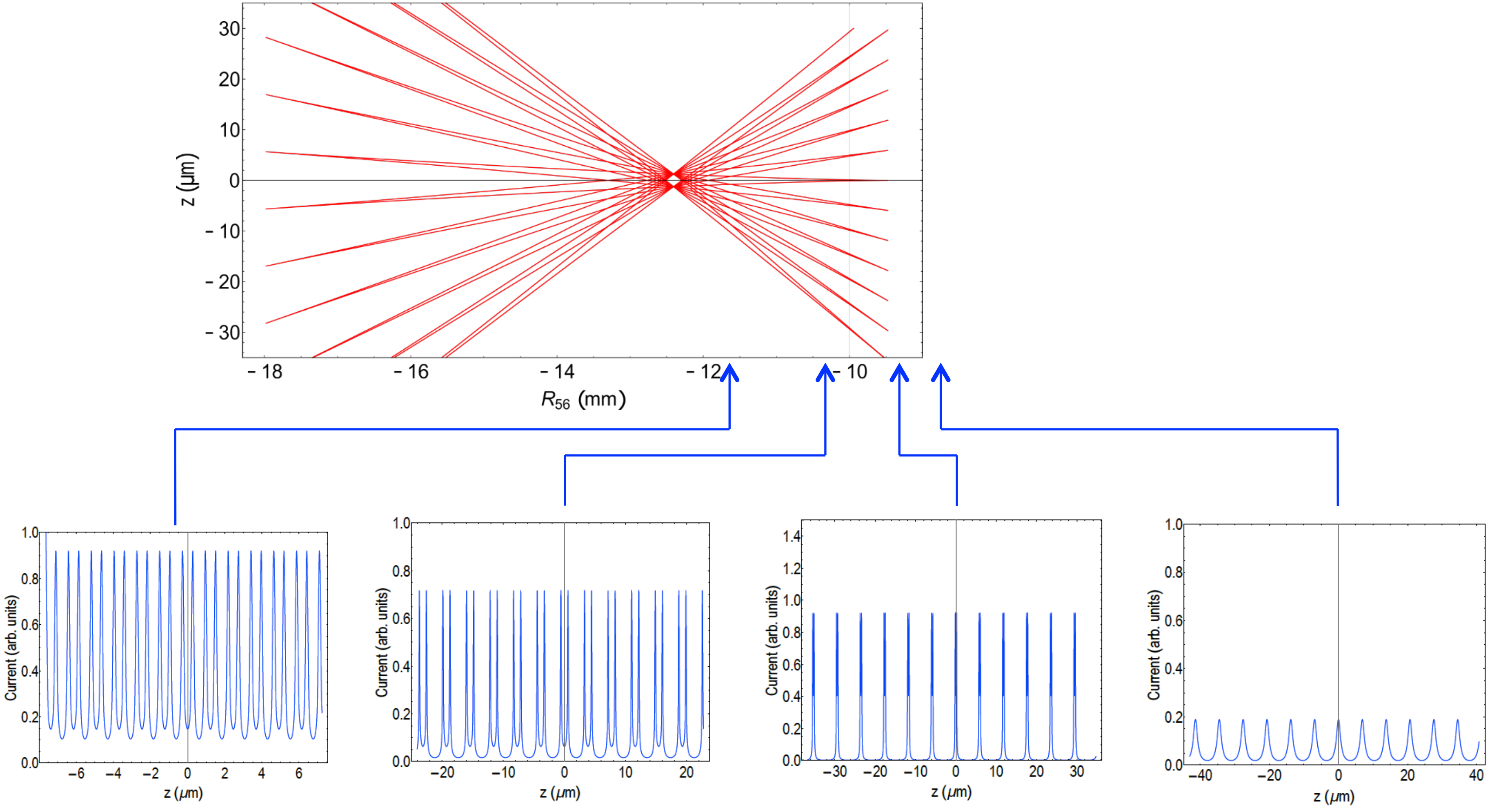
Microbunching is also a caustic phenomenon

- A caustic expression can also be derived for microbunching. Each line shows where a current spike will be seen.





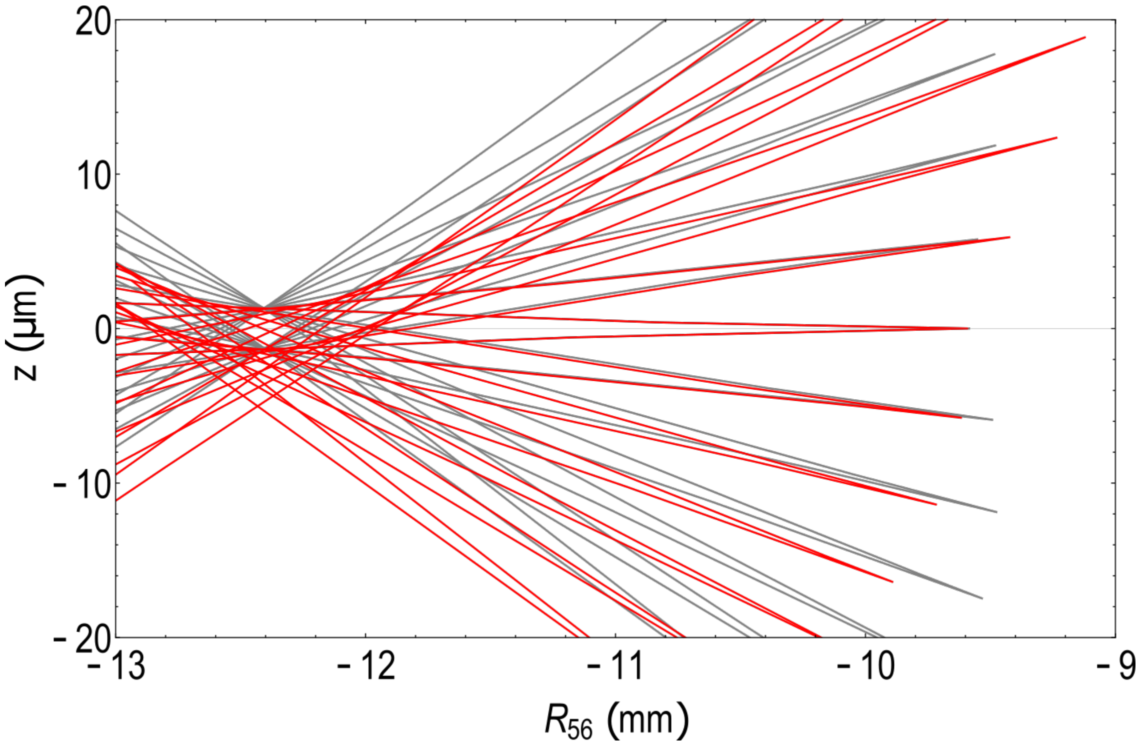
Microbunching is also a caustic phenomenon



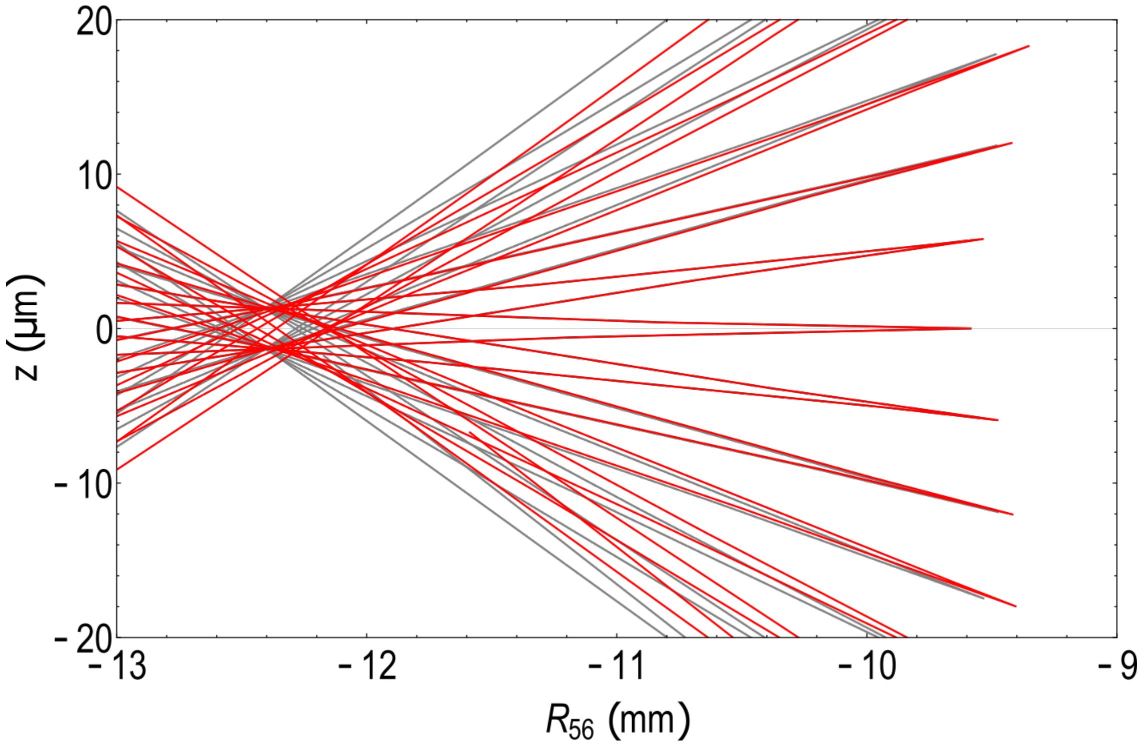
(d) $R_{56} = -11.8$ mm (c) $R_{56} = -10.5$ mm (b) $R_{56} = -9.5$ mm (a) $R_{56} = -9.0$ mm

Microbunching – with higher-order effects

- Including and varying higher-order effects we can see how the caustic lines change.



T566 = U5666 = 0 mm (gray)
T566 = -30 mm, U5666 =0 mm (red)



T566 = U5666 = 0 mm (gray)
U5666 = -2 m, T566 = 0 (red)

Conclusions

- Caustics can form in electron trajectories which are always associated with current spikes.
- Current horns from strong bunch compression can be avoided with higher-order magnetic elements.

I would like to thank:

- Rohan Dowd (Australian Synchrotron)
- David Paganin (Monash University)
- Andrea Latina (CERN)
- Mark Boland (Canadian Light Source)

The Ian Potter Foundation

Thank you

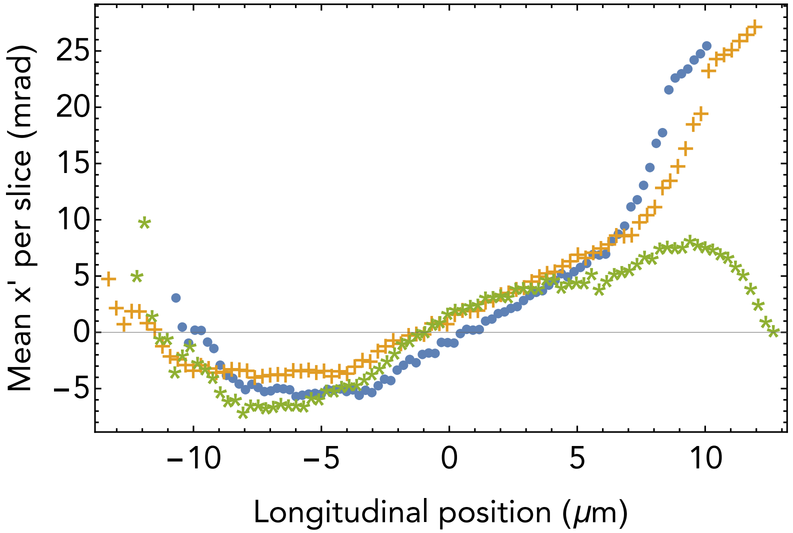
Back-up slides

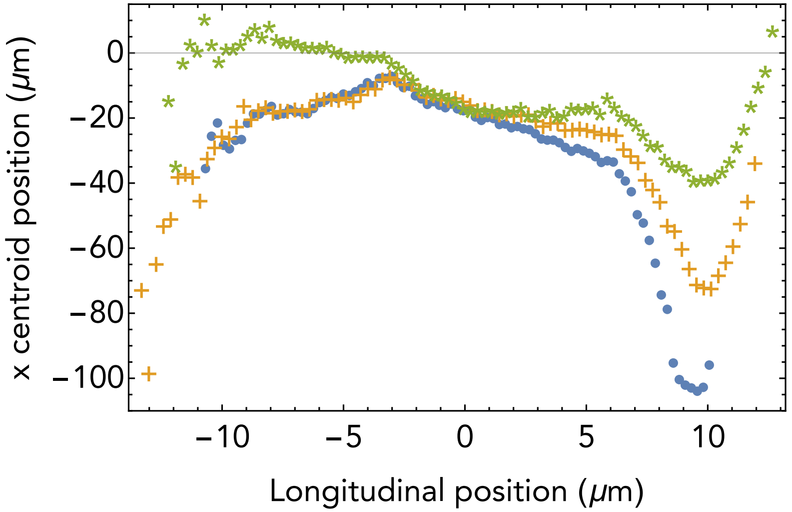
X-band linac

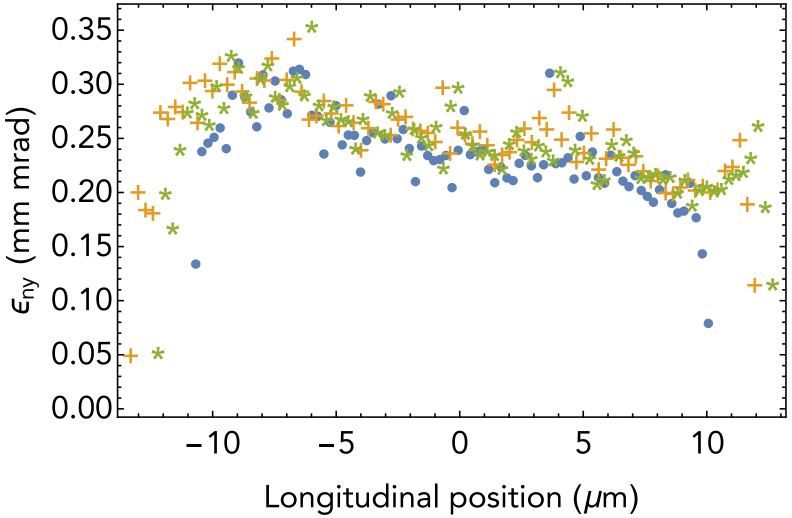
Without octupole: •

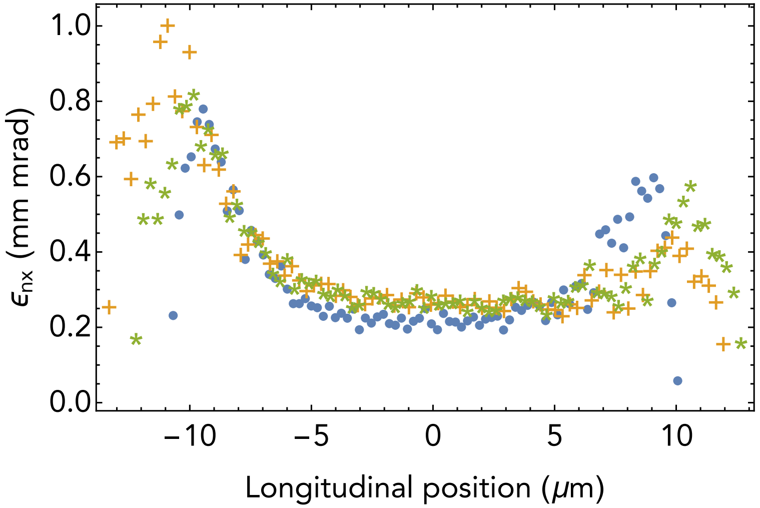
With octupole: +

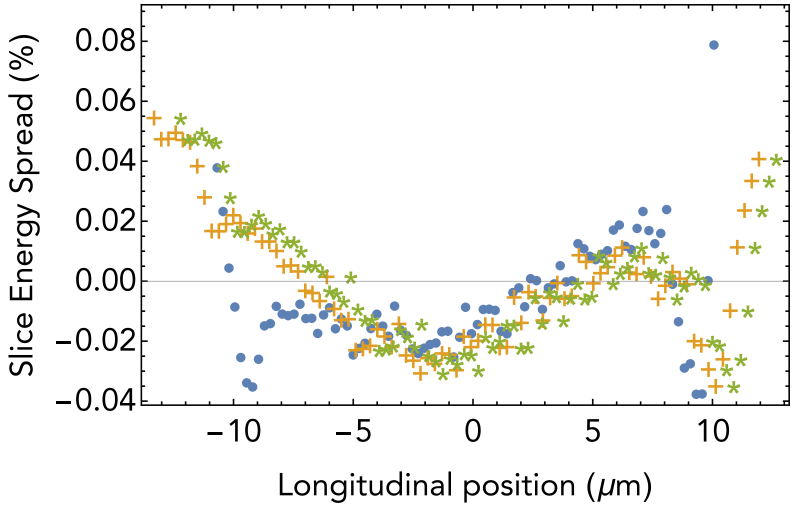
With octupole and sextupole: *











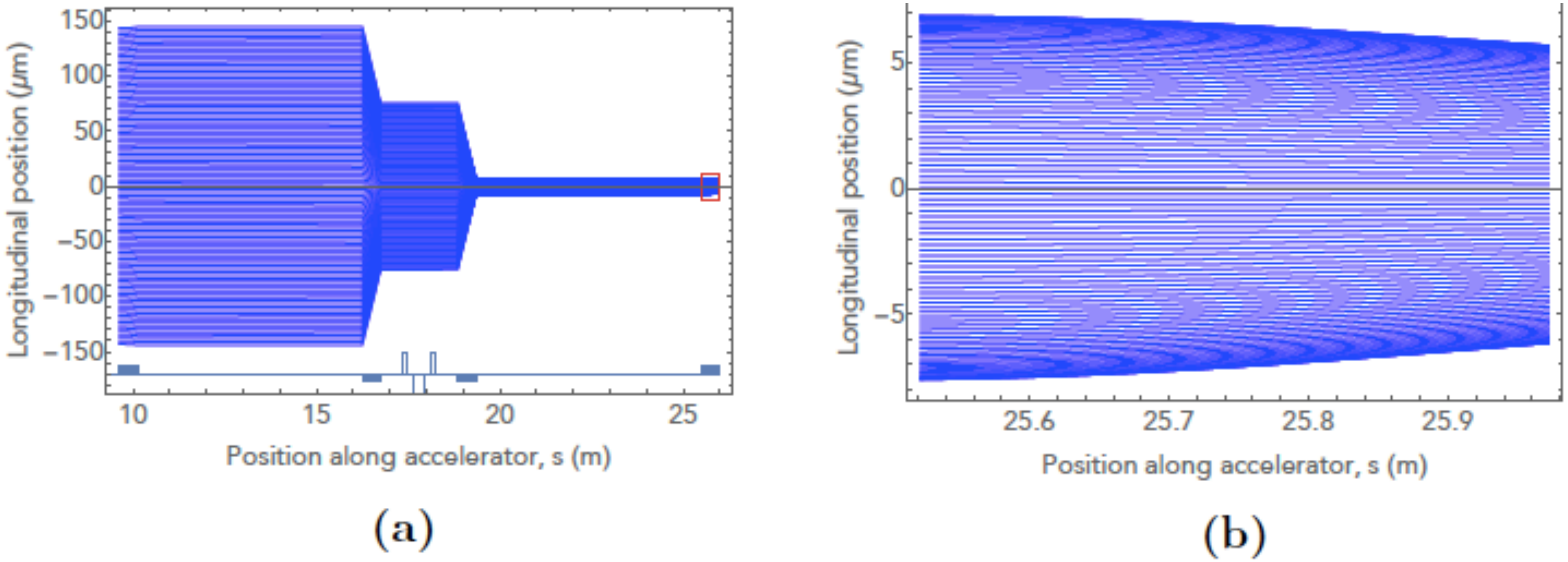
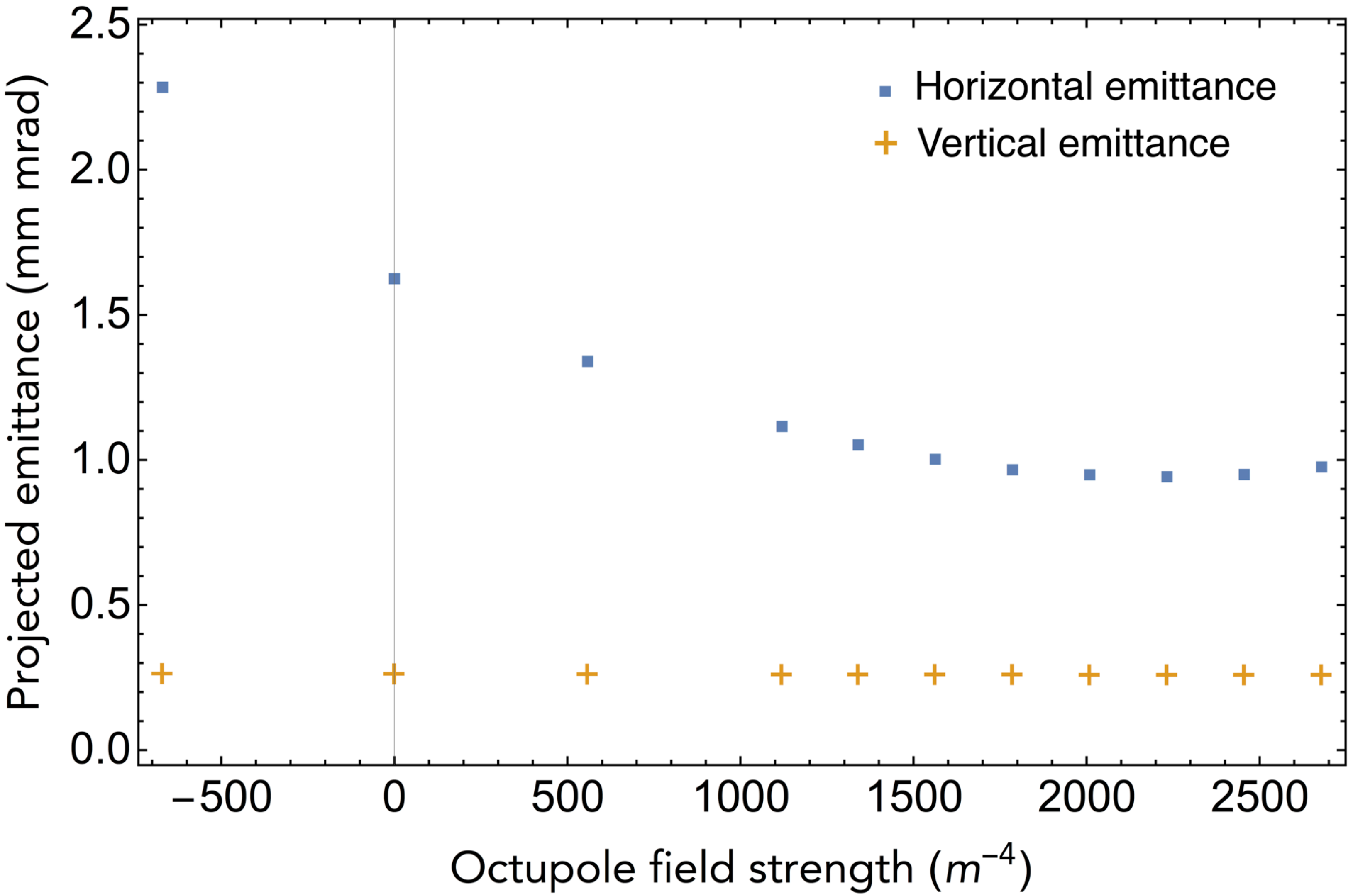


Figure 6.8: Electron trajectories (in s–z space) through a bunch compressor. (b) shows a close up of the trajectories through the fourth dipole (i.e. region outlined with a red box in (a)). Caustics can be seen forming at head and tail of the bunch in (b).

TABLE I: Beam properties at the end of the final linac section, for (a) Baseline layout, (b) Layout 1: which includes BC1 octupole magnet (Fig. 6a), (c) Layout 2 which includes BC2 sextupole magnet (Fig. 6b).

Parameter	Symbol	Units	Baseline	Layout 1	Layout 2
Bunch length	σ_z	μm	6.65	6.75	6.68
Horizontal bunch size	σ_x	μm	0.376	0.306	0.267
Vertical bunch size	σ_y	μm	0.161	0.162	0.163
Energy spread	$\sigma_{\Delta E/E}$	%	0.0371 (core)	0.0292	0.0281
Peak current	I_{peak}	kA	3.02 (core)	3.02	3.09
Total compression ratio	CR	-	121.38[a]	119.6	120.8
Bunch charge	Q	pC	250	250	250
Electron energy	E	GeV	6.16	6.16	6.16
Projected horizontal emittance	$\epsilon_{n,x}$	mm mrad	1.394	0.974	0.842
Mean horizontal slice emittance	$\epsilon_{s,n,x}$	mm mrad	0.386	0.392	0.377
Projected vertical emittance	$\epsilon_{n,y}$	mm mrad	0.274	0.273	0.274
Mean vertical slice emittance	$\epsilon_{s,n,y}$	mm mrad	0.255	0.249	0.246

[a] Note the bending angles of BC2 were reduced by less than 0.01% to bring the compression ratio down to be in-line with Layout 1 and Layout 2.



Horizontal emittance
Vertical emittance
Projected emittance (mm mrad)
2.5
2.0
1.5
1.0
0.5
0.0
−500
0
500
1000
1500
2000
2500
Octupole field strength (m^{-4})